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A POLAR CONTINUUM THEORY FOR SOLID CONTINUA

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Abstract

The Jacobian of deformation at a material point can be decomposed into the stretch tensor and the rotation tensor. Thus, varying Jacobians of deformation at the neighboring material points in the deforming volume of solid continua would yield varying stretch and rotation tensors at the material points. Measures of strain, such as Green's strain, at a material point are purely a function of the stretch tensor, i.e. the rotation tensor plays no role in these measures. Alternatively, we could also consider decomposition of displacement gradient tensor into symmetric and skew symmetric tensors. Skew symmetric tensor is also a measure of pure rotations whereas symmetric tensor is a measure of strains, i.e. stretches. The measures of rotations in these two approaches describe the same physics but are in different forms. Polar decomposition gives the rotation matrix and not the rotation angles whereas the skew symmetric part of the displacement gradient tensor yields rotation angles that are explicitly and conveniently defined in terms of the displacement gradients. The varying rotations and rotation rates arise in all deforming solid continua due to varying deformation of the continua at neighboring material points, hence are internal to the volume of solid continua and are explicitly defined by the deformation, therefore do not require additional degrees of freedom to define them.

Key Words : *Polar continuum theory, Polar decomposition, Lagrangian description, Internal polar theory for solid continua.*

If the internal varying rotations and their rates are resisted by the continua, then there must exist internal moments corresponding to these. The internal rotations and their rates and the corresponding moments can result in additional energy storage and dissipation. This physics is all internal to the deforming continua (hence does not require consideration of additional external degrees of freedom and associated external moments) and is neglected in the presently used continuum theories for isotropic, homogeneous solid continua. The continuum theory presented in this paper considers internal varying rotations and associated conjugate moments in the derivation of the conservation and balance laws, thus the theory presented in this paper is a polar theory for solid continua but is different than the micropolar theories published currently in which material points have six external degrees of freedom i.e. rotations are additional external degrees of freedom. This polar continuum theory only accounts for internal rotations and associated moments that exist as a consequence of deformation but are neglected in the present theories. We call this theory "a polar continuum theory" as it considers rotations and moments as conjugate pairs in a deforming solid continua though these are internal, hence are purely related to the deformation of the solid. In this theory we have no concept of 'stress couples' at the onset of the derivation of the theory. At this stage all we know is that internal varying rotations must be accounted for in a continuum theory for solid continua. It is shown that the polar continuum theory presented in this paper is not the same as the strain gradient theories reported in the literature. The differences are obviously in terms of the physics described by them and the mathematical details associated with conservation and balance laws. In this paper, we only consider polar continuum theory for small deformation and small strain. This polar continuum theory is suitable for isotropic, homogeneous solid matter such as thermoelastic and thermoviscoelastic solid continua with and without memory when the deformation is small. The paper also presents preliminary material helpful in consideration of the constitutive theories for polar continua.